Quest for Mathematics I (E2): Exercise sheet 3

1. On which interval from the selection $[-1, 0]$, $[0, 1]$, $[1, 2]$, $[2, 3]$ does the equation

$$-7x^3 + 20x^2 - x + 1 = 0$$

have a solution?

2. Let $T(\theta)$ be the temperature on the equator at longitudinal angle θ (measured in radians), and assume that T varies continuously with θ. By considering $f(\theta) = T(\theta + \pi) - T(\theta)$, show that there are two opposite points on the equator at the same temperature.

3. Are the following functions differentiable at $x = 0$?

   (a) $f(x) = \frac{x^2}{|x|}$

   (b) $f(x) = \frac{x^3}{|x|}$

   (c) $f(x) = x$ for $x < 0$ and $f(x) = x^2$ for $x \geq 0$

   (d) $f(x) = x$ for $x < 0$ and $f(x) = \sin x$ for $x \geq 0$

4. Compute the derivatives of the following functions (where they exist).

   (a) $f(x) = \sin(x^2)$

   (b) $f(x) = (\sin x)^2$

   (c) $f(x) = \frac{x^2}{7x^3+1}$

   (d) $f(x) = \sqrt{x+1}(4x^2 + 1)$

   (e) $f(x) = \sqrt[3]{x^3 + 1}$